

Polariton effects in optical spectra of microcavities

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The coupling between electronic excitations and optical modes have been of considerable interest in studies of atoms in optical cavities and optical excitations in semiconductor microcavities. The electromagnetic spectra of atoms in optical cavities exhibit splittings due to the coupling of the atomic dipole transitions with the radiation field [1, 2]. These coupled modes are called the vacuum field Rabi oscillations and the splittings between them are the vacuum field Rabi splittings. In case of electronic excitations in semiconductor microcavities the optical modes are coupled to excitons and the exciton-photon modes are often called cavity polaritons [3, 4]. These couplings are important for understanding such effects as optical bistability and laser action.

In many theoretical studies of coupled modes the electronic excitations were represented by a two-level system (a spin model). In this representation the higher lying states of the excitation are neglected. The difference in energy of the two levels is taken to be the excitation energy between the lowest two states of the atom, ΔE_a , or the excitation energy of an exciton in a semiconductor, $E_g - E_b$, where E_g is the band gap and E_b is the exciton binding energy. Systems with N two-level systems coupled to an electromagnetic mode with non-zero linewidth have been studied in the literature [5] where it has been shown that the Rabi splitting is proportional to \sqrt{N} . The linewidth of the optical mode represents, for example, the finite Q of the cavity.

Harmonic oscillators can also be used to represent the atomic excitations, and this representation offers greater ease of mathematical treatment than does the spin representation. The oscillator exhibits vacuum field Rabi oscillations when coupled to a quantized electromagnetic mode, although it does not exhibit driven Rabi oscillations in external fields [1]. Here we present a quantum mechanical treatment of the vacuum field Rabi splitting using an N harmonic oscillators model including finite linewidths of the electronic and optical modes. Actually, we consider two models. In the first one the interacting modes representing photons and excitons are each coupled to a source of dissipation [6]. In the second model the photon is taken to be coupled to a continuum of the electromagnetic states into which the cavity photon state can "leak", or tunnel [7, 8].

The Hamiltonian is that of the set of N harmonic oscillators with excitation energies ω_1 , all coupled to one oscillator representing the cavity photon with excitation energy ω_0 . In the resonant approximation, which is sufficient for ω_0 close to ω_1 , the Hamiltonian is given in terms of creation and annihilation operators as

$$H_{\text{osc}} = \hbar\omega_0 a_0^\dagger a_0 + \sum_{i=1}^N \hbar\omega_1 a_i^\dagger a_i - \frac{\hbar\alpha}{2} \sum_{i=1}^N (a_0^\dagger a_i + a_i^\dagger a_0) \quad (1)$$

where α is proportional to the oscillator strength of the electronic transition and depends on the cavity geometrical parameters [4, 8].

In the first model [6] each of the oscillators is coupled linearly to a loss mechanism giving rise to two damping coefficients, β_0 for the photon and β_1 for each of the electronic

excitations. To simplify the results, here we assume the near resonance condition $\omega_0 \approx \omega_1$. Following the procedure given in Ref. [6], we obtain $N - 1$ degenerate frequencies and two different resonance frequencies. The difference between the resonance frequencies is the Rabi oscillation frequency of the interacting system. It is given, approximately, by

$$\Delta\omega \approx \sqrt{\alpha^2 N - \frac{1}{4}(\beta_0 - \beta_1)^2}. \quad (2)$$

The Rabi splitting, on the other hand, is obtained from the position of the peaks in the optical spectrum and will be different from $\Delta\omega$ in Eq. (2).

We obtain the absorption spectrum as the imaginary part of the susceptibility which is evaluated as the Fourier transform of the dipole-dipole correlation function. The result is

$$\chi''(\omega) = \frac{N}{8} \frac{\beta_1(\omega - \omega_1)^2 + \beta_0\alpha^2 N/4}{[(\omega - \omega_1)^2 - \alpha^2 N/4 - \beta_0\beta_1/4]^2 + (\beta_0 + \beta_1)^2(\omega - \omega_1)^2/4}. \quad (3)$$

The absorption spectrum has a doublet lineshape and the Rabi splitting is given by the difference between the peaks. If the damping coefficients β_0 and β_1 are much smaller than $\alpha\sqrt{N}$ the widths of the two peaks are approximately $(\beta_0 + \beta_1)/2$.

In the second model [7, 8] we consider coupling of the cavity photon to the continuum of the electromagnetic excitations described by operator $b(\Omega)$ where Ω is the continuum of frequencies, $0 < \Omega < \infty$. The coupling of the cavity optical mode a_0 to b is given by

$$\int_0^\infty d\Omega [V(\Omega)a_0^+ b(\Omega) + V^*(\Omega)b^+(\Omega)a_0]. \quad (4)$$

The coupling of a_0 to N oscillators a_i is the same as in Eq. (1). We find that the absorption spectrum lineshape in this model has the same form as in Eq. (3) where photon linewidth, β_0 , is replaced by a term determined by V in Eq. (4).

The emission spectrum can be found by evaluating the distribution of energies in the spectrum of the outside photons, represented by operators b , in the time $t \rightarrow \infty$ limit. At $t = 0$ the cavity polaritons are assumed to have an equilibrium distribution at a given temperature [8]. In this way we obtain a frequency dependence of the emission spectrum which has a doublet lineshape. In the case of the laterally patterned semiconductor microcavities we have obtained the dependence of the Rabi splitting of the ground state polariton on the lateral size of a cavity studied in Ref. [4] and have found good agreement with the experimental results.

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